“Making Sense Differently”
Using Differentiated Instruction to Strengthen Students’ Number Sense

Rachel Tom
First Grade Intern
Ferguson Township Elementary
State College Area School District
rkt5009@psu.edu
# Table of Contents

Context ................................................................................................................................................. page 3

Wonderings ........................................................................................................................................ page 4

Data: Collection ................................................................................................................................. page 6

Data: Analysis

   Analysis Before Interventions .................................................................................................... page 10

   Analysis During Interventions ................................................................................................ page 11

   Analysis After Interventions .................................................................................................... page 12

Claims and Evidence ........................................................................................................................ page 14

Reflections and Future Practice ..................................................................................................... page 18

Index of Appendices ...................................................................................................................... page 21
During the 2009-2010 school year, I have been an intern in a first grade classroom at Ferguson Township Elementary, located in the State College Area School District. The structure of our day allows ample time for language arts and math, with one hour of writer’s workshop, one hour of language arts stations (word study, writing, guided reading, and listening), and one hour of math time. We also have a 35-50 minute block for science or social studies, depending on the unit content.

The class is comprised of twenty-four six and seven year olds, of which thirteen are female and eleven are male. The class represents a variety of socio-economic statuses, living situations, family structures, and racial, ethnic and cultural backgrounds.

The students also represent a variety of academic abilities. Weekly, four students receive math enrichment, three receive art enrichment, three attend Response to Intervention for language arts, and two attend learning support for language arts and math. Student reading levels, based on running records, are reflected by the benchmark books students should be able to read at certain points of the year. Six students are reading at the beginning of second grade, six are reading at the end of first grade, seven are reading at the February first grade benchmark, and five are reading below the February benchmark. Our most recent math assessment tested students’ understanding relating to our unit of study, Survey Questions and Secret Rules. Four students scored below basic, four students scored basic, six students scored between basic and proficient, nine students scored proficient, and one student scored between proficient and advanced.
Throughout the year, the students have grown as a community of learners. Most of our math, science, and social studies work is completed with a partner, fostering communication skills, creativity, and willingness to participate and share ideas. I believe that the students learn from one another each day, and that interacting with one another as they attempt to build new understanding is one of the most effective teaching strategies. Learning together also gives students ample opportunity to cultivate relationships and develop social skills, redefining their understanding of social cues, boundaries, and appropriate behavior. Each student is unique and brings something new and different to our classroom each day. (See appendix A for full inquiry brief and appendix B for annotated bibliography.)

Wonderings and Sub-Wonderings

Through the math program, Investigations, we began the school year with a two units, both focusing on building students’ number sense. John Van de Walle (2007) describes number sense as something that “develops as students understand the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as referents, and develop accurate perceptions about the effects of operations on numbers,” (p. 129). While students are expected to continue to develop their number sense throughout the year, I felt that in many cases, their lack of, or gaps in understanding hindered their success across many math activities.

I conducted two informal activities to gain a sense of my students understanding of two number sense components, place value and number relationships. Based on the
analysis of these two activities, I could now validate my hunch that my students had a wide range of understandings when it came to these topics. To determine whether or not this situation was unique to my classroom, I distributed a brief survey to the other teachers in our division, which includes two first and two second grade teachers. Based on their responses, I found that gaps in number sense often hinder a student’s performance.

(Teacher Surveys can be found in appendix C and Place Value Pre-assessment can be found in appendix D.)

As I continued to observe my students during math, their actions and thought processes led me to wonder to what extent a student’s lack of number sense will hinder their performance in other mathematical areas. Was there anything that I could do to address these needs?

**Main Wondering:**

- What effect will differentiated instruction have on specific areas of students’ number sense?

**Sub Wonderings:**

- What effect does differentiation have on students’ overall math performance?
- How does using students’ areas of strength support their number sense?
- How effective is using differentiating instruction to fill in perceived gaps in students’ understanding?
DATA: METHODS OF COLLECTION

I collected many types of data before, during, and after my interventions. Data collected prior to the interventions was used to determine specific areas of student need, as well as the learning modality, and in some cases modalities, that seemed to best support each student’s learning. Using this data, I was able to divide students into four groups, each with a specific focus related to number sense. During the interventions, I collected data through video, student work samples, and observational/anecdotal notes. These helped me determine students’ strategies and understandings as they became apparent to me through their actions and explanations of their thoughts. After the interventions, I collected several data samples to measure the effect of my interventions as well as changes in students’ strategies or understandings.

Pre-intervention data used to create differentiated groups:

- **Math interest/ability survey:** Students responded to the survey in small groups; I hoped to establish a baseline of students’ feelings about math, perceptions of their math abilities, and modality preferences during math activities. *(Student survey samples can be found in appendix E.)*

- **Patterned Set Recognition:** Students were shown a dot formation in standard form (as it would be seen on a die), or groups of dots. They were to determine and record how many dots they saw. I hoped to see which students could recognize certain formations quickly and mentally generate a total. This activity also confirmed my hunches about certain students’ strengths as visual learners. *(See appendix F for
examples of patterned sets presented to students and appendix G for student responses during the task.

- Dot formations – More, Less the Same: Given a dot formation, students had to draw a formation that was more, one that was less, and one that was the same (in amount). Students were also responsible for calculating how many more/how many less for the appropriate formations. I used this activity as an indicator of students’ sense of larger and smaller numbers, as well as their understanding of number relationships. (Student work samples can be found in appendix H.)

- Five Frames: This activity was used to support and assess students’ efficiency in writing combinations for the number five. After explicit modeling, students used manipulatives and a five frame on their worksheet to write as many number sentences that they could show using the frame. This allowed me to see how flexible and organized students were in their thinking, whether or not they used the five frame as I had demonstrated, and if they could come up all of the possible solutions. (See appendix I for power-point slides used to present the five frames, and appendix J for student work samples.)

- Number Sentences with Beans: Students used bean-shaped counters with one red and one white face to show and record number sentences with a sum of five. This helped show students’ flexibility, efficiency, use of a strategy, and utilization of an organizational strategy, if any. (See appendix K for student work samples.)

- Doubles Worksheet: The worksheet gave students explicit practice solving doubles (a number sentence with two of the same addends.) Students then listed the appropriate double needed for solving a separate addition problem. The worksheet
was useful in determining which students understood and could apply the strategy effectively, and which needed additional instruction. (See appendix L for examples of completed doubles worksheets.)

- **Dot Plates and Combinations**: The dot plates were used as an activity during math stations where I was able to supervise, model, and explicitly describe the process I wanted the students to follow. Each plate had dot stickers of two different colors on it. Students were responsible for using the three numbers (color 1, color 2, and total) to write number sentences. In this case, I told students at the outset that there were four possible number sentences that they could write. (See appendix M for student work samples.)

- **Fact Fluency**: Timed practice sheets were completed three times throughout the course of my inquiry. An untimed fact fluency test was administered once as part of a unit assessment. (For fact fluency samples see appendix N for timed and appendix O for untimed student work.)

- **Word Problem Post-Assessment**: I re-administered a unit assessment component to determine/measure what natural changes in number sense took place over time and before I began my interventions. The initial test was administered on December 19, 2009; it was administered the second time on April 6, 2010. (Samples of the re-administered assessment can be found in appendix P.)

**Data collected during interventions:**

- **Intervention matrix**: To help keep track of student progress and understanding, I organized my notes and observations from the recorded video footage of each group in a matrix. This data was helpful in making claims about individual students and
groups, and in identifying areas that needed attention in the future. *(See appendix Q for full intervention matrix.)*

- **Student work samples:** Throughout the intervention period, I was able to collect many artifacts. These samples varied according to group-specific tasks, and provided evidence to support my claims. *(See appendices R and S for student work completed during interventions.)*

- **Math Thinking books:** Students use a notebook to complete work that is part of the regular math instruction. These books and work completed after my interventions were used to make claims as appropriate. *(See appendix T for examples of student work as related to the regular math curriculum.)*

**Data collected after interventions:**

- **Re-administered math interest/ability survey:** The students completed the same survey, administered in small groups. The results were compared to initial student responses. *(See appendix F.)*

- **Re-administered doubles worksheet:** Student improvement and change in understanding was measured by comparing the before and after samples. *(See appendix L.)*

- **Fact fluency:** Students completed the addition fact fluency sheet; scores were compared to pre-intervention scores. *(See appendix N.)*
DATA: PROCESS OF ANALYSIS BEFORE INTERVENTIONS

My initial hunches about differences in student understanding prompted me to collect several pieces of data that demonstrated student capability and need in different areas of number sense. Although the before intervention activities differed, all were scored on the same basic system (See Figure 1). This helped me generate a profile of each student’s areas of understanding, misunderstanding, as well as the learning modalities that may be most supportive for that student. Based on this scoring system, I was able to examine my students’ work, looking specifically for strategy usage and accuracy. I also made many notes about the students’ use of strategies, whether they were approximating, misusing, or not using them at all. In many instances, I also made note of specific understandings that students demonstrated through their work on these tasks that helped better identify their needs as a learner.

Figure 1: Scoring System and Descriptions

<table>
<thead>
<tr>
<th>Advanced</th>
<th>Demonstrated ability to solve the problem using a higher-level strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proficient</td>
<td>Demonstrated ability to solve the problem using a basic strategy; may have missed one or two questions/components</td>
</tr>
<tr>
<td>Basic</td>
<td>Demonstrated attempt to use strategy or relied solely on prior knowledge; may have missed more than three questions/components</td>
</tr>
<tr>
<td>Below Basic</td>
<td>May have missed all or most questions/components and shows no sign of using a strategy</td>
</tr>
</tbody>
</table>

My next step was to use this data to group students for differentiated instruction. I used my own observations, their performance on modality-specific tasks, and their survey responses to create learning modality preference groups. The modality preferences, shown in Figure 2, were then linked with each student as they were grouped by demonstrated
area of need. The result was groups of students who shared the same needs as well as the same learning modality preference, shown in Figure 3.

Figure 2: Student Modality Preference

![Student Modality Preference](image)

Figure 3: Differentiated Groups

<table>
<thead>
<tr>
<th>Focus Area</th>
<th>Group Designation</th>
<th>Description of Differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Articulation</td>
<td>4</td>
<td>Using visual models and kinesthetic representations to help describe and explain thought processes</td>
</tr>
<tr>
<td>Articulation</td>
<td>3</td>
<td>Using a process oriented approach and organization to help check solutions and explain thought processes</td>
</tr>
<tr>
<td>Organization</td>
<td>2</td>
<td>Using visual organization to show relationships, study strategies, and build understanding of operations with the aid of manipulatives</td>
</tr>
<tr>
<td>Operational and Part-part-whole relationships</td>
<td>1</td>
<td>Using highly kinesthetic activities to develop understanding of part-part-whole relationships, the relationship between addition and subtraction, and developing efficiency using number combinations and other strategies with the aid of visual representations where appropriate</td>
</tr>
</tbody>
</table>

**DATA: PROCESS OF ANALYSIS DURING INTERVENTIONS**

Using my differentiated groups, I conducted interventions over a three-week period. The interventions took place in small groups outside of the classroom for ten to twenty minutes. Throughout my interventions, I conducted ongoing analysis by reviewing the footage taken during the interventions to determine what future steps were most
appropriate for each group. As I reviewed and entered anecdotal notes, observations, and video into my intervention matrix (appendix Q), I bolded evidence of what my next steps should be for each group. The matrix includes only interventions implemented within my specific groups, allowing me to focus solely on several minor aspects of number sense that were most appropriate for each group. I was also able to differentiate many curriculum tasks, though the majority of them did not relate specifically to number sense. These examples of differentiation are not included in my matrix, as they were not intended to act as interventions. Although I did not measure the effectiveness of these differentiation trials, I can comment on the usefulness of identifying students’ preferred learning modalities. This knowledge was essential as I differentiated our tasks throughout a geometry unit, allowing me to support and challenge all students appropriately.

**DATA: PROCESS OF ANALYSIS AFTER INTERVENTIONS**

As I reviewed the data collected before and during interventions, I began thinking about the claims I would be able to make specifically relating to the interventions conducted for each group. The claims I made are directly related to the interventions and subsequent groups affected. Thus, I have organized my process of data analysis after interventions based on how that data specifically relates to my claims.

- Data Analysis for Group 1
  - Because my interventions for this group focused specifically on number and operational relationships, I first focused on their fact fluency. Comparing the before and after scores revealed that practicing explicitly with numbers and
combinations yielded no significant change (an increase or decrease of more than four points.) Second, I reviewed the video footage and anecdotal notes taken during the intervention periods. I was able to identify the areas students continued to struggle with and make claims based on the way these claims related to the other data I reviewed. Finally, I used the before and after samples of the activity “More, Less, the Same.” Analyzing and comparing these two samples allowed me to measure the effect of the intervention. Based on this analysis, I was able to make Claim 1. *(Fact fluency samples can be found in appendix O and “More, Less, the Same” samples in appendix H.)*

- Data Analysis for Group 2
  - First I examined the before and after samples of the Doubles Worksheet. Doing so allowed me to see what effects the intervention and specific study of the strategy had on students’ performance and application. Additionally, I used the written response portion of the worksheet to help measure their understanding and ability to explain the strategy. A review of intervention footage of a final discussion of this strategy also allowed me to hear the students articulate their understanding of the strategy itself. I also used student work collected during the interventions to measure and study their application of the strategy. Based on this data, I was able to make Claim 2. *(See appendix L for examples of doubles worksheets and appendix Q for intervention notes. Student samples are found in appendix R.)*
I began analyzing data for Groups 3 and 4 separately, but found I was unable to generate a solid claim for each individual group. As noted in Figure 3, the focus area for both of these groups was the same while the differentiation was more specific to the modalities presented. To generate my claim, I looked at video footage of both groups playing a game called “Race to 100.” Students worked with a partner rolling one die, and added on each quantity until they reached 100. To keep track of their total, they used single cubes and ten sticks, regrouping as necessary. Additionally, I used work completed during several intervention periods that focused on word problems. I was able to analyze their strategies and apparent understandings using these samples, and compare them with my anecdotal notes/observations from the Intervention Matrix to help substantiate my claim. Finally, I looked at their Math Thinking books to further understand their thought processes, strategy application, and general understandings. Based on this data, I was able to make Claim 2. (Intervention notes can be found in appendix Q, student work samples in appendix S, and Math Thinking book samples in appendix T.)

CLAIMS AND EVIDENCE

Claim 1: Students who demonstrated difficulty understanding and explaining number relationships before interventions demonstrate difficulty during and after the interventions.
Evidence 1a: The before and after intervention timed fact fluency tests showed no noticeable or significant improvement in timed fact fluency scores (+/- four points.) On the untimed fact fluency test, only one student answered all questions correctly. The only similarity among the tests of the remaining five students was that they all missed 8+7. (See appendix O.)

Evidence 1b: Students were asked to compare various sets of dots and then record how many more and how many less dots as appropriate. On the “before” intervention sample, four out of six students recorded the total instead of “how many more,” and five out of six students recorded the total instead of “how many less.” On the “after” intervention sample, one student correctly answered both questions; one student answered only “how many less” correctly. The four remaining students recorded totals instead of answering how many more/less. (See appendix H.)

Evidence 1c: Anecdotal notes and observations from intervention footage highlighted students continuing difficulty with number relationships. In one activity, students were asked to build several different cube towers, adding on and removing cubes in response to my questions. Students were then shown a tower of five cubes. I asked if they could make a tower with two less cubes than this one; three out of the five students present made towers of two. Evidence 1b and 1c both demonstrate students growing and developing understanding of number relationships, and indicate that they have not mastered this concept. (See appendix Q.)
Claim 2: Students who received explicit practice and instruction related to the doubles strategy showed improvement in their ability to explain and apply the strategy but did not show an increase in choosing to use the strategy when given a choice.

Evidence 2a: When asked to complete the doubles worksheet before the intervention, students struggled when listing a double that would help solve a given addition problem. Some students were able to list a double, but may have listed the incorrect one; others were unable to list a double, and rewrote the given number sentence. After my interventions, which included explicit discussions of doubles as a strategy, four out of the six students were able to complete the worksheet with the correct double and solution; two students still struggled with the application of the strategy, and demonstrated similar struggles across multiple tasks. (See appendix L.)

Evidence 2b: Student work samples demonstrated the low-incidence of choosing doubles as a strategy within this group. Students were asked to solve the problem 7+6. Only one student used the doubles strategy, as demonstrated on her work sample. Two students in this group used tallies and three counted on with their fingers or in their head. (See appendix R.)

Evidence 2c: Students were given two opportunities to express their thoughts and understanding of the doubles strategy. Students were able to identify the usefulness of the strategy based on written responses on their doubles worksheet. A discussion during an intervention period yielded similar responses. (See appendices L and Q.)

Claim 3: Students who demonstrated deep understanding of place value demonstrated more efficiency and simplicity when solving other mathematical problems.
Evidence 3a: When playing the game “Race to 100,” students used single cubes, ten sticks, and a die to add on until they reached a total of 100. Three out of the twelve students involved were able to combine and regroup the tens and ones without taking the intermediary step of physically combining the two groups of addends, and then regrouping. For example, rather than adding four single cubes to a group of eight and then take ten away and replace it with a ten stick, these students would first remove the six single cubes away and then add a ten stick. (See appendix Q.)

Evidence 3b: During several interventions, we practiced solving a variety of word problems. The three students mentioned above demonstrated efficient mental strategies when solving their word problems. One student connected the problem 31-27 to 11-7 to reach the answer 4. Another demonstrated an efficient use of manipulatives; she solved the same problem by using four cubes to add on to 27, where her peers using manipulatives took 27 cubes and added on 4, rather than using the four as placeholders when counting on. A third student, who was given a multiple addend addition problem, solved it by adding on each quantity in his head and named the total. I prompted him to use another strategy, and he recorded a number sentence that reflected the problem. (See appendix S.)

Evidence 3c: Students were asked during regular math instructional time to solve the following problem: There are eight children on a bus. How many hands are on the bus? One student of the aforementioned three students solved the problem mentally, able to count by twos eight times, and recording the solution of sixteen. When prompted to check his work, he used tallies and a number sentence to reach the same solution. The other two students used a picture and wrote the total. Compared to the work of these three students, the other nine students required the use of combined strategies, most commonly a
numbered picture and number sentence, to solve the same problem. *(See appendix T.)*

**REFLECTIONS AND FUTURE PRACTICE**

This inquiry has been a full and enriching learning experience across many aspects of teaching and will be applicable in all of my future classrooms. Based on my findings, I will continue to use, modify, and investigate methods of differentiation across all subject areas. The benefits greatly outweigh the initial time and energy spent identifying student strengths and needs, as well as the time spent organizing and creating differentiated activities that are supportive, yet challenging, and engaging for all students. Towards the end of my inquiry, I felt that putting my methods of differentiation in place allowed me to use instructional time more efficiently and effectively.

Differentiation also kept students on-task, focused, and increased their independence as learners; they were often required to work independently or with a partner while I was conducting an intervention or leading a station with another group. This allowed me to explore what expectations, routines, and procedures one must have in place if students will be working independently. In the future, I will have to be certain that my students are ready for this responsibility and independence, and plan accordingly so that the learning environment is conducive to that success.

Differentiating also provided the necessary flexibility to modify tasks, groups, and learning modalities; however, I felt that differentiating was most useful when I could modify the tasks and re-design them to meet one particular need of a group of students. This allowed me to work with students who have a range of skills effectively and
appropriately, and support the child’s efforts and my own while working towards short and long-term learning goals. Overall, I see many uses for differentiation in my future classroom, and hope to be able to use differentiation in ways that are exciting, engaging, and effective.

During my final process of data analysis, I began realizing that many of my “predicted” outcomes of intervention were very different from the outcomes that I actually observed. I believed initially that my interventions would be able to fill certain gaps in students’ number sense. However, I feel that I have learned new approaches and practices that are effective, identified ones that are not, and overall, understand my students’ mathematical personalities on a much deeper level. Initially, I had hoped to learn what strategies were more effective for improving fact fluency. Instead, I learned that inquiry is meant to run its own course, not be guided towards fulfilling one’s hopes. For me, it was incredibly difficult not to conduct this inquiry based on my own premises and believed truths; in the future, I will have to make a conscious effort to conduct my inquiries more objectively, with more objective data samples and analysis techniques, and more flexibility to let the data take me wherever it will.

Learning the inquiry process was an immense part of the overall learning experience. I learned as I spent more time engrossed in the process that it can be consuming. In my future classroom, I hope to use inquiry in many ways, whether it is a small interest I have that relates to something happening in the classroom, or if there is a problem that I am hoping to address through an intervention that I can measure. I do not believe that every inquiry needs to be formal; it seems that teachers are doing many different types of inquiries and subsequent interventions/responses each day in their own
rooms. Knowing that I have the experience and the tools to conduct an inquiry gives me a feeling of competence and security, knowing that I will be able to meet the needs of my students through my own careful observation and consideration of the data I am collecting. I also learned about collecting data and the process of analyzing it and analysis throughout my inquiry. As an educator, I believe that this inquiry will help me use assessment data, student work, and my own observations/anecdotal notes more effectively. Based on what I find through careful analysis, I can make more informed teaching decisions that will support all of my students.

As a new teacher, life-long learner, and committed professional, I believe that this inquiry experience has helped me understand the importance of critical thinking and problem solving in my own classroom. It is important to use what data I have and what I can to make sense of what I do not understand, and feel comfortable using inquiry to analyze and evaluate my teaching practices and overall effectiveness. Inquiry-based learning is also an important component that I will include in my classroom; identifying an authentic problem, authentic data, and analyzing to draw conclusions is one of the greatest skills I can cultivate in my students. I hope that my future inquiries will help me design learning opportunities for my students that have been as rich and as meaningful as my own inquiry has been to me.
## APPENDIX

### Index of Samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Inquiry Brief</td>
<td>22</td>
</tr>
<tr>
<td>B. Annotated Bibliography</td>
<td>26</td>
</tr>
<tr>
<td>C. Teacher Surveys</td>
<td>33</td>
</tr>
<tr>
<td>D. Place Value Pre-assessment</td>
<td>37</td>
</tr>
<tr>
<td>E. Student Survey Responses</td>
<td>38</td>
</tr>
<tr>
<td>F. Patterned Set Activity</td>
<td>41</td>
</tr>
<tr>
<td>G. Patterned Set Recognition Student Samples</td>
<td>42</td>
</tr>
<tr>
<td>H. More, Less, the Same</td>
<td>43</td>
</tr>
<tr>
<td>I. Five Frames</td>
<td>44</td>
</tr>
<tr>
<td>J. Five Frames – Student Samples</td>
<td>45</td>
</tr>
<tr>
<td>K. Number Sentences with Beans</td>
<td>46</td>
</tr>
<tr>
<td>L. Doubles Worksheet</td>
<td>47</td>
</tr>
<tr>
<td>M. Dot Plates – Number Combinations</td>
<td>49</td>
</tr>
<tr>
<td>N. Fact Fluency – Timed</td>
<td>50</td>
</tr>
<tr>
<td>O. Fact Fluency – Untimed</td>
<td>51</td>
</tr>
<tr>
<td>P. Word Problem Post Assessment</td>
<td>52</td>
</tr>
<tr>
<td>Q. Intervention Matrix</td>
<td>53</td>
</tr>
<tr>
<td>R. Strategy Intervention – Student Samples</td>
<td>59</td>
</tr>
<tr>
<td>S. Articulation Intervention – Student Samples</td>
<td>60</td>
</tr>
<tr>
<td>T. “Math Thinking Book” Samples</td>
<td>65</td>
</tr>
</tbody>
</table>
APPENDIX A: INQUIRY BRIEF

CONTEXT

During the 2009-2010 school year, I have participated as an intern through Penn State’s Professional Development School. My placement in a first grade classroom in Ferguson Township Elementary School, located in the State College Area School District, has been an incredible experience, full of learning opportunities, professional development and a chance to try many out many teaching strategies and techniques. This year, we have twenty-four students total, thirteen girls and eleven boys. Each day we spend two full hours dedicated to language arts, where students are often grouped according to their reading and writing needs. In math, students most often receive whole group instruction, followed by work completed with a partner. Our science and social studies curricula are woven intricately throughout our language arts block, often using informational read-alouds and group projects to teach major ideas. We also have many discussions and brainstorm together, fostering an environment that is open to talking through one’s ideas and building conceptual understanding.

Our class is diverse in many ways, on many levels. The students’ personalities and quirks help to create a unique and diverse learning environment. Instructionally, students receive many supplemental services in addition to the regular classroom instruction. Weekly, four students receive math enrichment, three receive art enrichment, three attend Response to Intervention, a program which provides intensive, individualized, assessment-based remedial instruction, for language arts, and two attend learning support for language arts and math. These variations in learner needs are also reflected in the students’ performance thus far. Student reading levels, based on running records, are reflected by the books students are able to read and the books that they should be able to read at certain points of the year. Six students are reading at the beyond the June first grade benchmark to varying degrees, six are reading at the June first grade benchmark, seven are reading at the February first grade benchmark, and five are reading below the February benchmark to varying degrees. Our most recent math assessment tested students’ understanding relating to our unit of study, Survey Questions and Secret Rules. Four students scored below basic, four students scored basic, six students scored between basic and proficient, nine students scored proficient, and one student scored between proficient and advanced.

Not only do our students’ abilities vary, but their lives outside of school also play an important role in considering student needs. Ferguson Township Elementary School serves students from suburban developments, rural areas, trailers, and a variety of socio-economic statuses. Additionally, the students bring a number of different emotional, social, and behavioral needs to our classroom, which are addressed in multiple ways each day. Overall, our class has both strengths and weaknesses, but functions well together and throughout each day.

RATIONALE

Since beginning my internship, I have been intrigued with the math curriculum used in the district. Investigations is a hands-on program structured around activities that are meant to build conceptual understanding. The program for first grade involves a lot of partner work and games, each of which contributes to a small but significant portion of a
student’s understanding for a specific topic. What has been more interesting than Investigations itself, however, are the ways the students problem solve, use mathematical tools, such as manipulatives and drawing, and interpret mathematical questions. Their thought processes and actions during games and discussions have sometimes impressed me, baffled me, or both simultaneously. It became apparent for me early in September that first graders have very unique mathematical schemas and understandings of our mathematical world.

My intrigue was furthered with the help of a district-wide math training called “Gearing Up.” Every first grade teacher from the district gathered with the curriculum supervisors to receive training on new components of the curriculum and discuss ways that teachers could enrich the curriculum through a variety in task and differentiation. Each suggestion aimed at helping teachers understand their students’ thought processes and responding to them effectively.

However, my main inspiration in finding this inquiry was a prompt: “Think about a problem in your classroom.” It was easy for me; my biggest frustration in our classroom had been lack of student independence, specifically in math. Many of our students seem to rely heavily on being told how things work, rather than figuring it out through logic, problem solving, and utilizing prior knowledge. After some initial observation during the fall, I concluded that the variety of learners in our classroom made it such that a differentiated approach may be more efficient, effective and engaging for all students.

Wonderings

As I prepared to write this brief, I felt I already knew the answer to my question, “How can I help all students make gains in math?” The answer: differentiation. However, through several discussions, a review of many literary resources, and critical analysis of our students’ math abilities, I found myself wondering:

- What and when are the most effective ways to differentiate math instruction in my first grade classroom?
  - In what ways can I modify the math curriculum to help meet student needs? (I.e., variations in games)
  - In what ways can I modify the structure of our math time to help meet student needs? (i.e., small group instruction, tiered instruction, centers, independent work)
  - Will differentiation change students’ attitudes towards math?

Data Collection

I believe that I will have much data to collect throughout this inquiry. However, that will largely depend on my further refined and specific inquiry topic. As of right now, there are several methods of data collection that will definitely be incorporated into my inquiry. Data will be used during my inquiry to aid in the process of differentiation, as well as measure the effectiveness of the differentiation. The data collection methods are as follows:

- Previous unit assessment data: to identify strengths and struggles and create flexible groupings
Observations and Field Notes: To monitor student progress and thought processes. Observations and field notes will be useful in determining student needs as well as the effectiveness of differentiated instruction.

Surveys: Students will complete a pre and post survey about their math interest; data from these surveys will be compared to measure any changes that differentiation has caused in student attitudes towards math. Also, I will implement a self-assessment system; data from students will be used to create flexible grouping. Finally, surveys will be conducted throughout my inquiry to gain student feedback about the method of differentiation (if they liked it, if it was too hard, too easy, etc.)

Interviews (video-recordings): Using interviews and a set of questions or problems, I hope to identify student misconceptions, to gain insight into student thought processes, or measure the effectiveness of a differentiation strategy.

Student Work: Most student work will be documented through some type of checklist or assessment, possibly through field notes. However, specific student work samples can be compared to establish the effectiveness of differentiation and to further groups students flexibly to attain concepts.

**Timeline**

As of right now, I believe that my timeline will change significantly. This will depend largely on the topics I ultimately choose to focus on. However, the introduction of new units and outline for general data collection period and data analysis period should remain the same regardless of other changes that take place.

<table>
<thead>
<tr>
<th>Week</th>
<th>Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 2</td>
<td>• Administer interest survey</td>
</tr>
<tr>
<td>February 15</td>
<td>• Graph survey results to generate lists of students based on likeness of math, perceived difficulty, favorite method of problem solving, and preference for working alone/with partner</td>
</tr>
<tr>
<td></td>
<td>• Conduct field notes to determine students' level of understanding</td>
</tr>
<tr>
<td>Week 3</td>
<td>• Introduce differentiated fact families to students working on subtraction timesheets</td>
</tr>
<tr>
<td>February 22</td>
<td>• Differentiate geometry lessons based on field notes</td>
</tr>
<tr>
<td></td>
<td>• Continue to observe students (basis for next type of grouping)</td>
</tr>
<tr>
<td>Week 4</td>
<td>• Differentiate geometry lessons based on Week 3 data</td>
</tr>
<tr>
<td>March 1</td>
<td>• Introduce self-assessment technique</td>
</tr>
<tr>
<td></td>
<td>• Administer fact fluency time sheet</td>
</tr>
<tr>
<td></td>
<td>• Practice fact fluency through differentiation</td>
</tr>
<tr>
<td></td>
<td>• Observe to collect and analyze data</td>
</tr>
<tr>
<td>Week 5</td>
<td>• Differentiate geometry lessons based on Week 4 data</td>
</tr>
<tr>
<td>March 15</td>
<td>• Practice self-assessment</td>
</tr>
<tr>
<td></td>
<td>• Administer survey for feedback on differentiated lessons</td>
</tr>
<tr>
<td></td>
<td>• Administer fact fluency time sheet</td>
</tr>
<tr>
<td></td>
<td>• Observe to collect and analyze data</td>
</tr>
<tr>
<td>Week 6</td>
<td>• Differentiate geometry lessons based on Week 4 data</td>
</tr>
</tbody>
</table>
| March 22 | • Practice self-assessment  
• Administer survey for feedback on differentiated lessons  
• Administer fact fluency time sheet  
• Observe to collect and analyze data  
• **ON FRIDAY: Implement self-assessment and collect data; use data to format instruction for Week 7 |
| --- | --- |
| Week 7  
March 29 | • Differentiate geometry lessons based on Week 4 data  
• Practice self-assessment  
• Administer survey for feedback on differentiated lessons  
• Administer fact fluency time sheet  
• Observe to collect and analyze data  
• **ON FRIDAY: Implement self-assessment and collect data; use data to format instruction for Week 8 and collect data/student feedback on how they felt about the past week’s instruction |
| Week 8-Week 11  
April 5-April 26 | • Differentiate geometry lessons based on Week 4 data  
• Introduce new unit – word problems: pre-assess, observe and create initial groupings  
• Practice self-assessment  
• Administer survey for feedback on differentiated lessons  
• Administer fact fluency time sheet  
• Observe to collect and analyze data  
• **ON FRIDAY: Implement self-assessment and collect data; use data to format instruction for Week 9, 10 and 11; collect data/student feedback on how they felt about the past week’s instruction |
| Week 12-Week 14  
May 3-May 16 | • Post-survey  
• Continue to differentiate  
• DATA ANALYSIS  
• Write Inquiry paper  
• Prepare for inquiry conference |

**CONCLUSION**

I hope to find many answers to questions that extend beyond my wonderings throughout the course of this inquiry. Ultimately, I hope to find ways to implement differentiated instruction effectively, to use assessment to drive my instruction, and help my students each build their conceptual understanding of mathematics.
APPENDIX B: ANNOTATED BIBLIOGRAPHY


The article from The Access Center offered a brief overview of differentiated instruction, how differentiated instruction is implemented, a summary of differentiation techniques, and specific examples of how these techniques may be applied in math instruction. I found the examples of differentiated math instruction to be especially useful when thinking about how I might differentiate my instruction. Additionally, these examples helped me think about what structures of differentiation (i.e., tiered) and types of differentiation (i.e., visual, kinesthetic and auditory) might be most successful based on the specific needs of the learners in my classroom.


The Indiana Department of Education offered a list of PDF resource files of lessons that were tiered for math, science and language arts based on readiness, interest and learning styles. After reading many books and articles about what differentiation was and how to do it, exploring the IN DOE website was extremely helpful; I could see how all of the principles of differentiation look in a lesson plan. The examples of tiered instruction helped me connect the principles of tiering to practice, and implementing as parallel tasks. The other benefit of reading and reviewing these lesson plans were that they created tiered lessons for learning styles, which I was uncertain and hesitant to do myself. Seeing a lesson plan based on tiered learning styles gave me ways think about how I can tier my lessons for the learning styles in my classroom, a practice that I will definitely use throughout the course of my inquiry.


Bender’s book focused specifically on differentiating math instruction. The most useful components of this book were the sections in each chapter dedicated to “Teaching Tactics.” In these sections, Bender outlined tactics and specific strategies for different scenarios from developing number sense to teaching through multiple intelligences. Additionally, Bender’s chapter on Response to Intervention mathematics was also useful in framing my thoughts on differentiating for
struggling students. While only one student in my class attends RTI for math, I found the specific examples of what a teacher can do to differentiate extremely helpful. Bender also wrote a chapter on elementary and primary mathematics, listing examples of games, outlining a model of learner skills, and describing appropriate practices concerning prompting and wait time. Overall, this book is an invaluable resource that I will return to many times throughout the course of my inquiry.


This article provided examples and insight into making differentiation flexible and effective in each classroom. The teacher, Ms. Martez, highlighted used a system of ongoing assessment that mirrored the ways in which she differentiated for different student groups. This process requires students to be honest and forthcoming about their understanding, as well as cooperative when working in pairs or groups. Finally, Ms. Martez uses standardized assessment samples to help set individual goals with her students at conferences. The ideas and methods for differentiating instruction that Ms. Martez has found effective listed in this article are helpful in framing my inquiry processes, and I plan to utilize some of these ideas and methods in my own practice. These examples provided a good starting point for me as I experiment and create ways to differentiate my instruction for the purposes of my inquiry.


Carboni’s article on early number sense was helpful in defining the strategies students may use when solving a computational problem, how those strategies develop, and what other ways students might use/develop strategies. Carboni describes and defines “number sense,” which was helpful when thinking about what aspects of number sense I might target throughout my interventions. However, the most useful component of this article for my inquiry was the description of what number sense activities can look like every day, from games to visuals, Carboni suggested ways to give children practice that were not drills, and were not time consuming. I used many of her ideas and activities when developing my own activities to be used during interventions.

This newsletter offered examples of differentiation and tips to help teachers make differentiated instruction as effective as possible. The newsletter outlines what differentiation is and is not, the importance of assessment in driving decisions about differentiation, and organizational structures for differentiation. The organizational structures gave me some examples of ways that I could differentiate my math instruction, including using flexible grouping, tiered instruction, and centers. Additionally, the newsletter highlighted the importance of thorough planning, preparation and management strategies when incorporating differentiation into a classroom. In terms of my inquiry, I now have several ideas of differentiation structures that I can use, as well as the awareness to consider what management issues may arise, and structuring tasks to keep students engaged throughout the lesson.


Faulkner’s article in *Teaching Exceptional Children* gives a concise but highly informative review of the components of number sense, the important roles they play in learning mathematics, and the implications they have on math instruction. Not only does she outline the six main components of number sense and what it means for a student to understand them, she also highlights the importance of instructional practices and teacher background knowledge to support students growing acquisition and development of subsequent number sense skills. Faulkner also highlights the importance of proper articulation of mathematical terms and concepts on the teacher’s part; doing so will further support students and cultivate an ability to articulate mathematics on their own.


“Glass, Bug, Mud” is an article that describes one teacher’s efforts to differentiate her math instruction. The system of glass, bug, mud was a way for students to self-assess their own understanding of a mathematical concept, allowing their teacher to differentiate their instruction appropriately. Some of the methods used to differentiate were leveled task cards, peer tutoring, and small group instruction. Using this system, the students showed increased motivation and self-efficacy about their own math abilities, as well as an increased interest in math. This article will be most useful to me when I begin to design a process to differentiate my own instruction; the article includes a step-by-step thought process for teachers to use and consider when starting differentiation for the first time, or in a new content area. These tips will also help me establish a baseline and a basic structure in the process of differentiating, even if my methods of differentiation change.

Kameenui and Griffin’s research and proposal for examining basal mathematics programs is pertinent to my inquiry in several ways. First, the article helped me build an understanding of the components of contexts to consider when differentiating instruction (the task, learner, situation organizer, setting and materials). The article outlined several specific aspects of word problems that influence students’ ability to reason through them and solve them. These areas include question placement, extraneous information, syntactical structure, multiple steps, relationships to previous problem(s), order of numerical presentation, problem length, presence of diagrams and readability. This aspect of the article will help me when using word problems or a pre-assessment to identify some of the more specific needs or areas of struggle for the learners in my class, and subsequently differentiate as appropriate. Finally, this article offered several ideas to consider when thinking about my students’ metacognitive abilities and structures. Part of my differentiation will be based on my understanding of students’ thought processes and problem-solving strategies. Overall, this article provided me with several ideas to consider when differentiating my instruction and approaches, as well as when differentiating word problems.


Minott’s article offered many new perspectives to consider when planning and implementing differentiation. First, Minott highlighted the benefits of thoughtful and deliberate reflection as a tool for improving differentiated instruction. This prompted me to think about how I can systematically reflect on my differentiation practices as a way to help identify strengths, abnormalities, or specific characteristics that contributed to the outcome of my differentiation. Minott also suggests that reflective teachers can more effectively differentiate because they have considered multiple aspects of both instruction and the learner, not just the content. Finally, Minott discusses the reflective teachers ability to “frame” during instruction and assessment. In other words, reflective teachers are more able to assess and analyze the feedback they are receiving as they receive it, and determine what are the best immediate steps to take to reach a desired outcome.I think that Minott’s article has offered me some “food for thought.” Now I am not only conscious of this proposed importance of reflection, but also curious as to how it might improve my own techniques.

The Morales, Shute and Pellegrino article summarizes several of the basic challenges that children face when encountering word problems. Some of these challenges include transferring their knowledge of number sense and operations to a new form (the word problem), reasoning through knowledge of one-to-one correspondence and part-whole relationships, and recognizing the differences in conceptual structures for different types of word problems. This article has helped me establish a better understanding for the many components and thought processes that are required to complete a word problem. Additionally, the article will be useful as a guide to additional ways that I may try to differentiate word problems for my students. Finally, the article provides an extremely informative classification table that describes each type of word problem and relates the problem to a broader, underlying conceptual structure. This table will also be helpful in planning my differentiation, as well as in the classification of my own word problems.


The Mathematical Thinking blog was a useful and informative resource that helped me better understand what activities can help a child’s number sense and in what ways. For example, I was able to look at many activities that would help students understand part-part-whole relationships, activities that would help students with strategies, activities that would help students with the relative size of numbers, and more. Having a variety of activities that related to different components of number sense was useful when it came to differentiating my tasks based on student need and struggle. Finally, I enjoyed reading and planning to implement these new and creative activities; as I attempted to complete thorough interventions, having creative tasks that helped engage and motivate students definitely supported my efforts.


This article on formative assessment is helpful in outlining the use and benefits of assessment to guide teaching practices. This article highlights the use of assessment to identify student understanding, clarifying what comes next in the learning process, allowing teachers to create intervention plans based on specific
student needs, improve instructional practices of teachers or teams, and motivate students, building confidence in themselves as learners. The use of formative assessment in the classroom can help students understand the goals they are working towards and be their own advocates when they are not meeting standards or progressing as they would like to. This article helped me understand that when differentiating instruction for my students, it will be important to be in touch with their learning needs and to support them as best as I can to reach the desired goals. Finally, the article stresses and gives characteristics of high quality assessment, which will be important in my inquiry as well as for any differentiation that I use in the future.


Tomlinson’s book offered a very thorough background on what differentiation is, why teachers differentiate, and how to differentiate. One of the most useful components of the book was the description of hallmarks and elements of a differentiated classroom. For my inquiry to be successful, classroom climate, teacher understanding of learner needs, learner willingness to take risks, teacher commitment to high standards, and additional factors must work cohesively. Tomlinson’s book offered an overview of all of these components, which I can now thoughtfully consider as I prepare to differentiate my instruction. Tomlinson also offered examples of the ways in which a teacher can differentiate, and what she might be responding to as she differentiates in these ways. Finally, Tomlinson outlined other differentiation structures and how to organize and implement them. These examples, such as centers and tiered instruction, gave me enough background information and understanding to decide how to use them in my classroom. After reading this book, I feel that I have a much better idea of what differentiated instruction actually is, how to go about differentiating, what to differentiate, and how to determine whether or not my differentiation is effective.


This article was a short summary that highlighted and described some of the key elements of differentiated instruction. Tomlinson offers ways for teachers to differentiate, whether it is through their content, process, or product, as well as ways to make the differentiation successful. Additionally, she provides justification for the importance and practicality of differentiation at the elementary level. This article is a resource that will help me understand the idea of differentiated

Van De Walle’s text on mathematics instruction was an appropriate resource for me in that it specifically discussed differentiating math instruction and creating problem-based learning environments. Van De Walle highlights that problem based learning experiences help students build their own understanding of a concept, and that appropriate scaffolding becomes an important component of problem-based learning for students who may struggle with a task. Additionally, Van De Walle outlines how flexible grouping allows teachers to ensure that students are receiving appropriate instruction based on their needs per specific content areas. The text also highlights assessment. Frequent and diagnostic assessment is a key component of both math and differentiated instruction. The text continues to outline several strategies for assessing students during mathematical activities, many of which I plan to use in my inquiry. Finally, the third chapter reviewed discusses the importance of teaching mathematics equitably, something I am passionate about. To help teachers teach math equitably, chapter six outlines several strategies for helping all students make gains, from peer-tutoring to alternative assessments. Overall, the Van De Walle text will be a critical resource throughout the inquiry process, and has been immensely helpful in guiding my preparation for data collection and differentiation techniques.


Yatvin's book on differentiated instruction provided a unique look at what I did not find in many of my other resources. One element of differentiation that Yatvin highlighted was the use of time and space. Although other resources highlighted that time and space are components of differentiation, Yatvin conveys the importance of having enough time to allow for students to engage in multiple activities that may vary in length, and having classroom spaces that are conducive to centers, group work, conferencing with students, and moving around fluidly. Additionally, Yatvin highlighted self-management as an important component of inquiry, using routines and signals, rules, procedures, and devices (i.e., records) to help tasks run smoothly. She also highlights the importance of meaningful and diagnostic assessment, offering many helpful examples and strategies of how to use these assessments to guide future instruction. She also makes the connection between using time differently and making more time to assess; this will be important for me to consider since I plan to incorporate more assessment into my instruction.
APPENDIX C: TEACHER SURVEYS

Grade:  2

Do you administer fact fluency practices of any kind?  Yes  No

If yes, please briefly describe:  
1st part of the yr. 10 prob. in 1 min.  
2nd part 80 prob. in 3 min.

-- Where do you perceive your students struggling with number sense?

___ I do not perceive my students struggling.

___ Relationships between different numbers (1 is less than 2)  
___ Patterned sets (Do not recognize common representations of a number, i.e., a die)

___ Part-part-whole (Do not understand that numbers are made up of two or more parts. 
I.e., 5 is 4 and 1, 2 and 3, 0 and 5.)

-- What, if any, strategies do your students use when solving addition and subtraction problems?

___ Benchmark or “nice” numbers (i.e., 5 and 10)  
___ Counting on or counting back

___ Doubles or near doubles

-- Do you feel that your students’ number sense hinders their success in other math tasks?

___ Yes  No

If yes, please explain:  Word prob. or algorithms w/larger 
numbers/double digit addition.


Additional comments about your students’ number sense, what you do to enhance students’ 
number sense, or anything else you’d like to add!

___ Whole activities/games that help children
___ “see” the relationship w/ + and –

THANK YOU 🙏
Grade: 1 2

Do you administer fact fluency practices of any kind? Yes  No
If yes, please briefly describe: time tests, fact strips, and computer programs

-- Where do you perceive your students struggling with number sense?

_____ I do not perceive my students struggling.

_____ Relationships between different numbers (1 is less than 2)

_____ Patterned sets (Do not recognize common representations of a number, i.e., a die)

✓ Part-part-whole (Do not understand that numbers are made up of two or more parts. I.e., 5 is 4 and 1, 2 and 3, 0 and 5.)

-- What, if any, strategies do your students use when solving addition and subtraction problems?

_____ Benchmark or “nice” numbers (i.e., 5 and 10)

✓ Counting on or counting back

✓ Doubles or near doubles

-- Do you feel that your students’ number sense hinders their success in other math tasks? Yes  No
If yes, please explain: their ability to problem solve and count coin combinations are hindered

Additional comments about your students’ number sense, what you do to enhance students’ number sense, or anything else you’d like to add!


THANK YOU ☺
Do you administer fact fluency practices of any kind? [Yes] No
If yes, please briefly describe: weekly computer work (timed one) +
monthly paper timed quizzes

-- Where do you perceive your students struggling with number sense?
   V I do not perceive my students struggling.
   ___ Relationships between different numbers (1 is less than 2)
   ___ Patterned sets (Do not recognize common representations of a number, i.e., a die)
   ___ Part-part-whole (Do not understand that numbers are made up of two or more parts.
   i.e., 5 is 4 and 1, 2 and 3, 0 and 5.)

-- What, if any, strategies do your students use when solving addition and subtraction problems?
   V Benchmark or "nice" numbers (i.e., 5 and 10)
   V Counting on or counting back
   V Doubles or near doubles

-- Do you feel that your students' number sense hinders their success in other math tasks?
   [Yes] No
   If yes, please explain: Sometimes in combining higher numbers, they could make more connections than they sometimes do.

Additional comments about your students' number sense, what you do to enhance students' number sense, or anything else you'd like to add!

The quick surges
Do you administer fact fluency practices of any kind?  
Yes  No

If yes, please briefly describe:  
We rotate through the computer fluency on FileMaker Pro each morning. We also do paper and pencil each week.

-- Where do you perceive your students struggling with number sense?

I do not perceive my students struggling.

Relationships between different numbers (1 is less than 2)

Patterned sets (Do not recognize common representations of a number, i.e., a die)

Part-part-whole (Do not understand that numbers are made up of two or more parts. I.e., 5 is 4 and 1, 2 and 3, 0 and 5.) This is probably the area some struggle with.

-- What, if any, strategies do your students use when solving addition and subtraction problems?

Benchmark or “nice” numbers (i.e., 5 and 10)

Counting on or counting back

Doubles or near doubles

-- Do you feel that your students’ number sense hinders their success in other math tasks?
Yes  No

If yes, please explain: ________________________________

______________________________

______________________________

Additional comments about your students’ number sense, what you do to enhance students’ number sense, or anything else you’d like to add!

______________________________

______________________________

______________________________

THANK YOU ☺
Given a model showing the hundreds, tens, and ones place, students had to roll three dice and use the digits to make the largest number possible. The line through the middle of each sample represents where students switched from making the largest to making the smallest number possible.
APPENDIX E: MATH SURVEY, FEBRUARY AND APRIL

A. 4/10

Name: 

1. I like math: 😊 😊
2. Math is: easy hard
3. I like to: use tools draw do math in my head
4. I like: working alone with a partner

B. 2/10

Name: 

1. I like math: 😊 😊
2. Math is: easy hard
3. I like to: use tools draw do math in my head
4. I like: working alone with a partner

C. 2/10

Name: 

1. I like math: 😊 😊
2. Math is: easy hard
3. I like to: use tools draw do math in my head
4. I like: working alone with a partner
Students completed this survey in small groups under teacher guidance. The A, B, and C, sample indicators match the students' response for February to their response in April. For question three, "using tools" may also be defined as using manipulatives. "Draw" may be defined as pictorial or symbolic representations. "Do math in my head" may be defined as mental math. Results of February survey listed below.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I like math:</td>
<td>☀ ☯</td>
</tr>
<tr>
<td>2. Math is:</td>
<td>easy hard</td>
</tr>
<tr>
<td>3. I like to:</td>
<td>use tools draw do math in my head</td>
</tr>
<tr>
<td>4. I like:</td>
<td>working alone with a partner</td>
</tr>
</tbody>
</table>

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I like math:</td>
<td>☀ ☯</td>
</tr>
<tr>
<td>2. Math is:</td>
<td>easy hard</td>
</tr>
<tr>
<td>3. I like to:</td>
<td>use tools draw do math in my head</td>
</tr>
<tr>
<td>4. I like:</td>
<td>working alone with a partner</td>
</tr>
</tbody>
</table>

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I like math:</td>
<td>☀ ☯</td>
</tr>
<tr>
<td>2. Math is:</td>
<td>easy hard</td>
</tr>
<tr>
<td>3. I like to:</td>
<td>use tools draw do math in my head</td>
</tr>
<tr>
<td>4. I like:</td>
<td>working alone with a partner</td>
</tr>
</tbody>
</table>
Math Survey Results
February 20, 2010 (Blue)
April 22, 2010 (Purple)

Enjoyment of Math

Math Ability – Student Perceptions of the Difficulty of the Subject

Work Preference
This activity was intended to identify which children responded well to highly visual tasks. Students saw the image on a screen for about three seconds, and then were required to record the total number of dots (Appendix G).
APPENDIX G: Student Responses in Patterned Set Activity

Correct answers: 1) 3, 2) 5, 3) 8, 4) 6, 5) 5, 6) 9, 7) 6, 8) 9
APPENDIX H: More, Less, the Same

Given a formation of dots (left), students had to draw three subsequent sets: one that had more dots, one that had the same number of dots, and one that had less. Then, they responded to the questions below.
I designed this powerpoint to present the five frame and how to use it.
Students were asked to show different combinations of five using beans (manipulatives) to fill in the spaces of the frame. This activity was implemented to identify students’ understanding of part-part-whole relationships.
APPENDIX K: Number Sentences with Beans

Students were asked to use beans (manipulatives) with a red and white face to create combinations for the number six. This activity had a concept focus of part-part-whole relationships, and a kinesthetic modality focus.
APPENDIX L: Doubles Worksheet

Let's practice our doubles:

1+1 = 2  
2+2 = 4  
3+3 = 6

4+4 = 8  
5+5 = 10  
6+6 = 12

7+7 = 14  
8+8 = 16  
9+9 = 18

Write a double to help you answer each number sentence. Then, write the answer.

<table>
<thead>
<tr>
<th>NUMBER SENTENCE</th>
<th>WHAT DOUBLE WILL HELP ME?</th>
<th>ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>4+3 = ?</td>
<td>4+3 = 7</td>
<td>7</td>
</tr>
<tr>
<td>5+6 = ?</td>
<td>5+6 = 11</td>
<td>11</td>
</tr>
<tr>
<td>3+2 = ?</td>
<td>3+2 = 5</td>
<td>5</td>
</tr>
<tr>
<td>3+5 = ?</td>
<td>3+3 = 6</td>
<td>6</td>
</tr>
<tr>
<td>1+2 = ?</td>
<td>1+1 = 2</td>
<td>2</td>
</tr>
<tr>
<td>4+5 = ?</td>
<td>4+4 = 8</td>
<td>8</td>
</tr>
</tbody>
</table>

How are doubles helpful when practicing math facts?

It makes math faster.

MAR 26 2010

Name:

4/23/10
Samples A from March and April are samples from the same student. Sample B is from a different student, one that continued to struggle with the strategy application after explicit interventions.
Given a plate with different colored dot stickers, students had to write number sentences using each of the numbers in that fact family. Sample A represents a student who has a working understanding of fact families and can apply that to this activity. Sample B represents a student who has a working understanding of combinations of a number, but not specifically how to use fact families.
### APPENDIX N: Fact Fluency, Timed

<table>
<thead>
<tr>
<th>Level 5 Addition Facts 1-10</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 2 4 + 1 7 + 3 5 + 2 4 + 4 8 + 7 3 + 6 0 + 5 3 + 1 3 + 2 7 + 0 6 + 3 9 + 5 4 + 6 8 + 3 9 + 6 8 + 7 8 + 5 6 + 2 8 + 1 2 + 0 1 + 9 8 + 1 1 + 0 10</td>
<td></td>
</tr>
<tr>
<td>6 5 7 8 4 8 8 8 9 3</td>
<td></td>
</tr>
</tbody>
</table>

Students were given three minutes to complete as many problems as they could. This fact fluency test was administered three times during my inquiry.
APPENDIX O: Fact Fluency, Untimed

Students were allowed an unlimited amount of time to complete this fact fluency assessment.
Students completed this portion of a previous unit assessment to measure changes in understanding that took place over three months without explicit instruction.
## APPENDIX Q: Intervention Matrix

<table>
<thead>
<tr>
<th>Date</th>
<th>Group/ Focus</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/14</td>
<td>C - Race to 100</td>
<td>Showed proficiency in adding amounts – added total number of cubes and then substituted. Only AV made direct substitutions (did not put ones in and regroup). In discussion, I found it difficult to express this concept; students seemed confused when asked to make a substitution without adding all of the ones. Even when we talked about knowing 8+4=12, students could not take away 7 ones and add one ten stick to make 12. Had to add the 4 and regroup. As the process oriented group, I tried to walk them through it mentally. <strong>Making next steps more process oriented by providing mental math structures for them to use?</strong> Step 1 – solve mentally, step 2 – decide how many 10s/1s, step 3 – show by adding a ten and taking away the appropriate number of ones. Requires them to think of multiple combinations of numbers when adding/taking away appropriate cubes. <strong>What is a better way to show this visually? Is there a better model?</strong></td>
</tr>
<tr>
<td>4/14</td>
<td>D – Race to 100</td>
<td>CA, AHl can make trades. Need to be more explicit with the rest – seem to understand the concept of regrouping/trading, but do not make the jump to automatically substitute to create the total (still adding the ones and regrouping.) Difficulty across the board articulating. <strong>Need to talk more explicitly about tens and ones, practice mental math to solve and simplify the process. Need more time to play/practice game – high interest. Short but explicit session necessary.</strong></td>
</tr>
<tr>
<td>4/14</td>
<td>D – Crayon Puzzles</td>
<td>Allowed independent time to work. I told them there was only one right answer for the first problem, since we had spent so much time working with combinations. Many had begun writing combinations already before I clarified. After, they seemed confused that there was only one right answer, so we read the problem again. Most produced the answer without the use of manipulatives or drawing; when asked, most responded with a variation of “they just knew.” It was difficult to get them to explain how they knew. One struggled. We re-read the problem and I asked what the important information was. (6 total, some red some blue, equal number of each). Together we used cubes to show making the total with the same red as blue. I modeled explicitly, but the students seemed to follow. Struggler seemed to understand just after showing one red/one blue, checking if they were equal, if we had some of each, and thinking about how we could get six all together. Subsequent puzzles will be more challenging, but students seemed relatively equipped to solve the first two.</td>
</tr>
<tr>
<td></td>
<td>A –</td>
<td>Used manipulatives and gave different scenarios of how many</td>
</tr>
</tbody>
</table>
Dinner Word Problems

Pieces of food are on each students’ plate. Who has more – students answered successfully. How many more? One student knew immediately. Let’s look at cubes. Students stacked cubes. TH – How could you tell how many more he has? “He has more because he has seven and she has six.” (Missing the how many more part). Asked students to use cubes and break the stick to show. Student breaks into 3 and 4 cubes. One student agrees. Ask student to write another number sentence; rephrase: how can we show how many more X has than Y? Start with the number 5. Students pass notebook and write the number sentence. Got stuck on completing the number sentence. How could we figure it out – what strategy? Suggested that we count by twos.

Able to identify who has more and how many more when cubes were in separate trays. Can articulate somewhat. Writing number sentences much more difficult.

Struggle with combining and separating and showing with manipulatives. Use different language? How can I be more clear?

Who thinks we’re combining? (Everyone). Why do you think we’re combining? Because you have the most and B has the least. In response, demonstrates confusing how many more with how many all together. I ask, How many in all? Student gives number sentence. Why am I going to add 6 and 3? Explicit. Because I want to know how many there are all together, I have to add and the red and the blue to get the total.

Student writes 3+3+3=8. How could I check my work? Students are confused – think it’s eight. Counted by threes, regrouped to make 4+4+1. Student suggest counting by ones, I name as counting all. How many are there all together? 9. Student asked to fix number sentence attempts to correct the addend. Review the two combinations. Who can make a new combination? 2+2+2+3 and 4+5

4/15 A – Station Time

Can you show two sticks – one with four, one with six? Can you write the number sentence? How many more are in the six stick? Six. (Others look confused, cannot produce an answer). Let’s look at this together. Six has two more than four. How can you tell? Because the stick of six is taller than the stick of four.

When asked to make a stick that is two less than their stick of five, three students made sticks of two. When reviewing, students justified a stick of three because they knew 3+2=5, and 5-2=3

4/15 B – Crayon

Group solved easily; did not use manipulatives; several drew representations. One student was stuck. Made two sticks of 10.
<table>
<thead>
<tr>
<th>Date</th>
<th>Grade</th>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/16</td>
<td>D – Word Problems</td>
<td>Focus was on mental math strategies using place value to solve problems with larger numbers. Initially, only two students went straight to work. The other four were thinking, watching, then began writing. These four then took manipulatives after recording the important numbers on their pages. One student solves problem with cubes; does not regroup the ten stick; comes up with 31-27=10. Emphasized explaining thought processes; FM demonstrated using two tens and eleven ones; counted on with cubes to keep track. AHl able to articulate what this student demonstrated. CA student does not regroup, but “covers” the necessary cubes on the ten stick, and shows the difference. NP – counted on in his head AHl relates 7+4=11 to 27+4 For the second word problem, students used a variety of strategies; linking numbers, number sentences with tens and ones, mental strategies, tally marks, etc. <strong>Asking clarifying questions prompts students to better explain their strategies and computation.</strong> Multiple strategies that require place value understanding and regrouping; some demonstrated proficiency and strategies that did not require regrouping or a thorough understanding of place value. (IB made two lines of 11, each with one cube and a ten stick. Covered up three on one line and seven on the other. Counted to get 7+8=15)</td>
<td></td>
</tr>
<tr>
<td>4/16</td>
<td>C – Word Problems</td>
<td>GP: given subtraction, uses total cubes and removes appropriate number, counts all. NR: Used cubes to compare the difference; made a line of twelve and a line of eight; counted how many more eight was than twelve. WR: Organized in standard pattern formation; added ones, then added tens.</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>Activity</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>-------------</td>
<td></td>
</tr>
</tbody>
</table>
| 4/16  | A – Crayon Puzzles | AV: with multiple addend addition, added on in his head each time
AH: counted back, got wrong answer, then drew picture.
SG: Repeating subtraction problem to find total. Wrote numbers and crossed off; got the right answer but recorded. When using cubes, realizes she miscounted. Practiced working backwards to check answer, justifies the new answer.
Focused on using manipulatives and checking our work first, before we recorded. Had students re-read the problem to check and see if their answer was logical. TW thinks he has six (has twelve.) Required that they have some blue and some red and that they have the same number of each. TH makes the same number of each, but no longer has six. TW does six of each. Convinced he has six total; we count all. Explicitly walking him through the problem. Three students are able to come up with combinations of seven, with more blue than red.
Going over multiple times, explicitly rereading directions, asking students to check their work to see if it matches. |
| 4/19  | B – Crayon Puzzles | What would be the first thing we need to think about? Deliberate question asking – attention problems. Redirecting. Repeating with the steps. Talking about combinations, listing all of the different strategies. After supporting students through listing combinations, I asked for what we need to do first. How many do we need to have total? (Prompting the students for each next step). Redirect. How many total? The same number of blue and red. Redirect. Review combinations. Choose one that has four total, same number of blue and red.
Word problem. (7+6). Wanted to get them to use doubles. Students work independently. Talking about strategies – how we used a picture, count on in head, double.
Another problem. 11 cats are in the yard. 4 go inside. 2 dogs come outside. How many cats are outside? (Trying to get them to distinguish between important/unimportant information.) Working through takes a long time – one states confusion. Repeat problem 3 times as students work. I ask students after working for a minute whether or not I need the information about the dogs; one is quick to say no and tell why “you’re only asking about the cats.” Three students struggle; we use cubes. What strategy did we use to figure it out? We counted. Using the different colors helped them count only the “Cats” and not the total (cats and dogs). Have students
restate why we only needed to count the cats.

<table>
<thead>
<tr>
<th>4/20</th>
<th>C – Mental Math Word Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8+3 = 11 Shows the parts with cubes, practice regrouping. Group of 8, group of 3. Students count total. Can we make some trades? “I’m going to trade ten cubes for one ten stick. And I’m going to keep the one, and I have eleven”</td>
</tr>
<tr>
<td></td>
<td>7+6 = 13. Students solve quickly mentally. Two students show the two groups. How could we regroup? Take ten cubes away and trade. Number sentence for total of 13.</td>
</tr>
<tr>
<td></td>
<td>4+8=12 One student thinks 15. Each student shows the two parts in cubes. Model for students. I know 8+4 is 12, and I know there is one ten and two ones in the number 12. Instead of counting out ten, could I just leave two ones and replace it with a ten stick? Is that the same? (Students look somewhat confused, seem to be building this connection.)</td>
</tr>
<tr>
<td></td>
<td>9+9=18 I demonstrate with cubes. I know there are nine and nine, so what might be an easy way to show this sum in cubes? Going straight from mental math to the solution.</td>
</tr>
<tr>
<td></td>
<td>Practiced tens and ones. Solve and tell me how many tens and ones there are. 22+11=33. Student demonstrates using cubes. Used tens and ones to show the separation of each place.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4/20</th>
<th>Group B – Five and ten frames</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students roll a die and show the face of the die and what it would look like on a ten frame. What happens if we have more than five? Why do we need to add another row? Students seem to get this without struggle. Practiced adding our rolled numbers together. How could we combine? What is it going to look like? Students used “standard” formations. Students take turns showing the number in more than one way. Able to do this quickly and successfully. Practiced with combinations.</td>
</tr>
<tr>
<td></td>
<td>Mental Math: 7+4. Student says she counted on in her head. 3+9. Student counts on in her head.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4-21</th>
<th>Group C – Cube Mysteries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Guessing combinations of cubes based on combinations. Trying to get students to think through what cubes might be hidden and ask probing questions. Students are quick to analyze what information they gather after each question. Some are stronger than others at deducing what the combination could be. Students sometimes ask around in circles. Not all follow the questioning and deduce further.</td>
</tr>
<tr>
<td></td>
<td>What made it easy? I could guess random things and figure it out. What was hard? The total, trying to figure out the numbers of blue and red.</td>
</tr>
<tr>
<td>Date</td>
<td>Group</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>4/21</td>
<td>Group B – Doubles</td>
</tr>
<tr>
<td>4/21</td>
<td>Group D – Cube Mysteries</td>
</tr>
</tbody>
</table>
APPENDIX R: Student Work Samples, Strategy Intervention

Sample 1

3 + 1 = 4
2 + 2 = 4
4 + 2 = 6

7 - 6 = ?
6 + 6 = 12
6 + 7 = 13
Candy

11 - 4 = 7
Sample 2

3 + 1 = 4
2 + 0 = 4
4 + 0 = 4

7 + 6 = 13
6 + 7 = 13

1 + 7 + 2 = 10

11 cats
4 cats inside
2 log cats outside
Student work represents the following:

- Crayon Puzzle: I have four crayons. Some are blue and some are red. I have the same number of each. How many of each could I have?
- Word Problem: I have seven pieces of candy. My mom gives me six more pieces of candy. How many candies do I have altogether?
- Word Problem with Distracter: There are eleven cats in the yard. Four cats go inside. Two dogs come outside. How many cats are outside?
APPENDIX S: Student Work Samples, Problem Solving Strategies Articulation

\[
\begin{array}{c}
27 & 31 & 4 \\
\hline
14 & 19 & 33 \\
\hline
22 & 3 & 4 & 15 \\
\end{array}
\]

\[
14 - 6 = 8
\]
14 cookies 19 Isabella Browens

Ben 3

22

3 + 22 = 25

4 - 19 = 15
Students were given one of the following problems. The problem letter corresponds with the student work sample letters.

**Mental Math Word Problems**  
April 16, 2010

A, B, C: A and N are going to trade baseball cards. A has 27 and N has 31. How many more baseball cards does N have?

D. A is playing Hide-and-seek with her cousins and brothers. There are 8 kids all together. A found five. How many are still hiding?

E. G has 14 baseball hats. He gave six to a friend. How many does he have left?

F. A is reading books about Pittsburgh and Philadelphia. He read two books about Pittsburgh, three books about Philadelphia, and two books about traveling. How many books did A read?
APPENDIX S: Student Work Samples – Word Problem from Regular Math Curriculum
These samples represent student responses to the following question:

There are eight children on the bus. How many hands are on the bus?